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The effect of numerical dissipation on the predictive accuracy of wall-modelled large-eddy simulation

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Abstract. The effect of numerical dissipation on the predictive accuracy of wall-modelled large-eddy simulation is investigated via systematic simulations of fully-developed turbulent channel flow. A total of 16 simulations are conducted using the open-source computational fluid dynamics software OpenFOAM®. Four densities of the computational mesh are considered, with four simulations performed on each, in turn varying in the amount of numerical dissipation introduced by the numerical scheme used for interpolating the convective fluxes. The results are compared to publicly-available data from direct numerical simulation of the same flow. Computed error profiles of all the considered flow quantities are shown to vary monotonically with the amount of dissipation introduced by the numerical schemes. As expected, increased dissipation leads to damping of high-frequency motions, which is clearly observed in the computed energy spectra. But it also results in increased energy of the large-scale motions, and a significant over-prediction of the turbulent kinetic energy in the inner region of the boundary layer. On the other hand, dissipation benefits the accuracy of the mean velocity profile, which in turn improves the prediction of the wall shear stress given by the wall model. Thus, in the current framework, the optimal choice for the dissipation of the numerical schemes may depend on the primary quantity of interest for the conducted simulation. With respect to the resolution of the grid, the change in the accuracy is much less predictable, and the optimal resolution depends on the considered quantity and the amount of dissipation introduced by the numerical schemes.

Keywords: large-eddy simulation; turbulence; wall modelling; numerical dissipation; channel flow.

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Влияние численной диссипации на расчетную точность метода моделирования крупных вихрей с пристенным моделированием

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Аннотация. В данной работе систематически исследуется влияние численной диссипации на расчетную точность метода моделирования крупных вихрей с пристенным моделированием. С этой целью в свободном программном обеспечении OpenFOAM® было проведено шестнадцать расчетов развитого турбулентного течения в канале. Расчеты проведены на сетках четырех разных плотностей, по четыре расчета на каждой сетке, в каждом из которых, в свою очередь, установлен разный уровень численной диссипации посредством изменения интерполяционной схемы для конвективного переноса. Проведено сравнение результатов расчетов с данными прямого численного моделирования, находящимися в открытом доступе. Показано, что профили ошибки всех рассмотренных величин находятся в монотонной зависимости от объема численной диссипации. Диссипация предсказуемо приводит к подавлению высокочастотных флуктуаций скорости. Кроме этого, она также приводит к увеличению энергии крупномасштабных флуктуаций и существенной переоценке уровня кинетической энергии турбулентности во внутреннем слое. Однако повышенный уровень диссипации приводит и к улучшению точности расчета средней скорости течения, что, в свою очередь, обеспечивает более точную оценку касательного напряжения на стенке пристенной моделью. Таким образом, оптимальный уровень диссипации может зависеть от основной цели расчета. Эффект плотности расчетной сетки на точность расчета трудно предсказуем, и оптимальное значение плотности зависит как от рассматриваемой физической величины, так и от уровня диссипативности интерполяционных схем.

Ключевые слова: метод моделирования крупных вихрей; турбулентность; пристенное моделирование; численная диссипация; течение в канале

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1. Introduction

In turbulent boundary layers (TBLs), two fundamental length-scales can be distinguished. One is the thickness of the boundary layer, δ, which governs the size of the largest eddies in the TBL. The other is the viscous length-scale, δ_ν, defining the size of the small eddies present in the inner region of the TBL. In standard, or wall-resolving, large-eddy simulation (LES) both scales are resolved, leading to an accurate solution, but forcing the size of the computational mesh to scale as $Re_\tau^{1.85}$, where $Re_\tau = \delta/\delta_\nu$ is the friction-based Reynolds number. In wall-modelled LES (WMLES) a model for the scales $\sim\delta_\nu$ is employed, allowing to use a mesh that only resolves the large scales $\sim\delta$. This reduces the mesh size scaling with Re_τ to linear, making higher Re_τ-number simulations affordable [1]–[3].

Several approaches to WMLES exist, the reader interested in a detailed review is referred to [4]–[7]. Most commonly, so-called wall-stress modelling is used, in which the task of the wall-model is to predict the wall shear stress τ_w, given the current solution to the LES equations, typically sampled from a single point located at some distance h from the considered location on the wall.

The predictive accuracy of WMLES has been assessed in numerous works, using both in-house codes and publicly available solvers, such as OpenFOAM. Most of the studies consider a single set of modelling choices (subgrid scale model, numerical methods, etc.) and focus on evaluating or improving the performance of the wall model, see, for example, [8]–[15]. However, a recent study [16] has shown that the other parameters of the simulation affect the accuracy of WMLES at least as much as the wall modelling. In particular, the role of numerical dissipation in the simulation was highlighted, with more dissipative schemes and subgrid scale models, somewhat surprisingly, leading to more accurate results for certain flow quantities. The goal of this work is to further analyze the role of numerical dissipation via a set of simulations in which the resolution of the computational mesh and the dissipative contribution of the numerical scheme used for interpolating convective fluxes are systematically varied. Both temporal statistics of the quantities of interests and energy spectra are considered, contributing to a more holistic picture of how numerical dissipation affects the solution.

2. Methods of computational fluid dynamics

The governing equations for LES are obtained by applying a spatial filter to the Navier-Stokes equations, see [17] for details. The resulting system of PDEs must be complemented with a model for the subgrid stresses. In this work, the WALE model [18] is used to that end.

To solve the governing equations, the open-source CFD software OpenFOAM v1806 is employed. OpenFOAM uses the finite volume method to discretize the governing equations, supports arbitrary convex polyhedral cells, and offers a rich selection of numerical schemes [19]–[21]. For wall modelling, an additional publicly available library is used to enhance OpenFOAM's built-in capabilities [22]. A particularly important improvement (see [23]) is that the library allows sampling the LES solution used as the input to the wall model from an arbitrary distance from the wall, and not only from the wall-adjacent cell.

In scale-resolving simulations, it is common practice to employ at least second-order accurate numerical schemes. Here, a fully implicit second-order accurate backward differencing scheme is used to integrate the equations in time, and linear interpolation is used to compute the diffusive cell-face fluxes. The choice of the scheme for interpolating the convective fluxes is used as a controller of the amount numerical dissipation in the simulation. To that end, a linear blending of two second-order schemes is considered: second-order upwind and linear interpolation. The former scheme is dissipative, and its weight, expressed in percent, will from here-on be referred to as "the amount of upwinding" for simplicity. In OpenFOAM, the scheme using 25% upwinding is referred to as LUST (linear upwind stabilized transport). WMLES comparing LUST and linear interpolation with no upwinding have been conducted in previous studies [16], [22], with more favorable results for first-order statistics achieved using LUST. Here, the cases of 15% and 5% upwinding are additionally considered to get a clearer picture of how upwinding affects the results.

An algebraic wall model based on Spalding's law [24] is employed.

3. Case set-up

Fully-developed turbulent channel flow at $Re_b = 125000$ is considered, where $Re_b = U_b\delta/\nu$, with $U_b = 1$ m/s denoting the bulk velocity, $\delta = 1$ m the channel half-height, and ν the kinematic viscosity. These values correspond to those used in the direct numerical simulation (DNS) of Lee and Moser [25], which is used here as reference data, with respect to which the relative errors in the simulation results are computed. The computational domain is a box of size $9\delta \times 2\delta \times 4\delta$ in the streamwise, wall-normal, and spanwise direction, respectively. The value of U_b is enforced at each time-step by a time-varying source term in the streamwise momentum equation.

The domain is discretized with cubic cells of equal size. The mesh is thus fully defined by the number of cells discretizing the channel half-height, denoted n/δ. Four meshes are considered, with n/δ equal to 15, 20, 25, and 30. For each mesh, four simulations are performed, corresponding to a different amount of upwinding employed in the convective flux interpolation scheme: 25%, 15%, 5%, and 0%, respectively.

The location of the sampling point of the wall model is set to $\approx 0.1\delta$ in each simulation. The approximation sign is necessary because 0.1δ does not coincide with a cell center for all the four considered mesh resolutions. However, the accuracy of the law of the wall is quite robust with respect to change in h, therefore comparing results across different mesh resolutions should still be possible.

4. Results

An analysis of the performance of the wall model is given first. The solid lines in fig. 1 show the relative error in the predictions of the mean friction velocity, $\langle u_\tau \rangle = \sqrt{\langle \tau_w \rangle / \rho}$. Both plots present the same data, but with a different quantity used as the abscissa. In the left plot, a trend towards underpredicting $\langle u_\tau \rangle$ with decreasing amount of upwinding is clearly observed. In the right plot, a similar trend is observed with respect to increasing the resolution of the mesh. The dashed lines in Fig. 1 show the relative error in the mean velocity at the location of the sampling point. The magnitude and behavior of this error are very similar to that exhibited by $\langle u_\tau \rangle$. This confirms the observation made in a previous study [16] that in the case of attached boundary layers the accuracy of the wall model is chiefly determined by the accuracy of the velocity signal that it bases its predictions on.

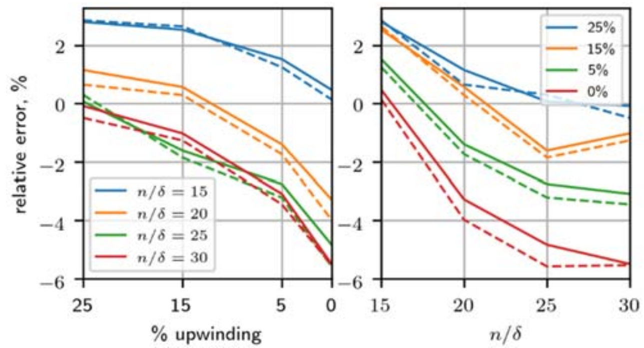

Fig. 1. Relative error in the mean friction velocity (solid lines), relative error in the mean velocity at the sampling point (dashed line). The same data are shown in both plots, with a different quantity used as the abscissa

Attention is now turned to the analysis of the mean velocity profiles in outer scaling, $\langle u \rangle / U_b$. The relative errors in the obtained values are shown in fig. 2. Only the values above the overlap region are plotted, since this is the region where we expect WMLES to give accurate results. It is observed that the amount of upwinding defines the behavior of the error. Less upwinding leads to increasingly larger under-prediction for $y \lesssim 0.5\delta$, and, consequently (due to a fixed flowrate), an over-prediction in the region above. Arguably, the best accuracy across all four considered mesh resolutions is obtained using 15% upwinding. Unfortunately, results generally do not improve with increased mesh resolution apart for the case with 25% upwinding.

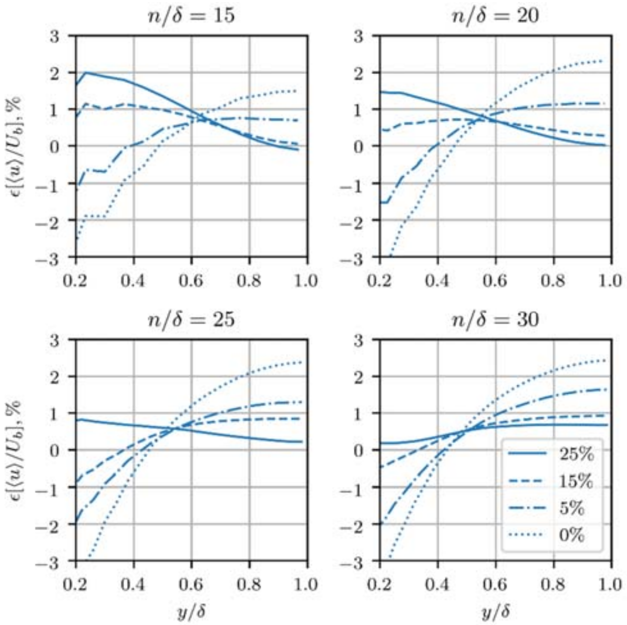

Fig. 2. Relative error in the outer-scaled mean velocity profiles as a function of y/δ

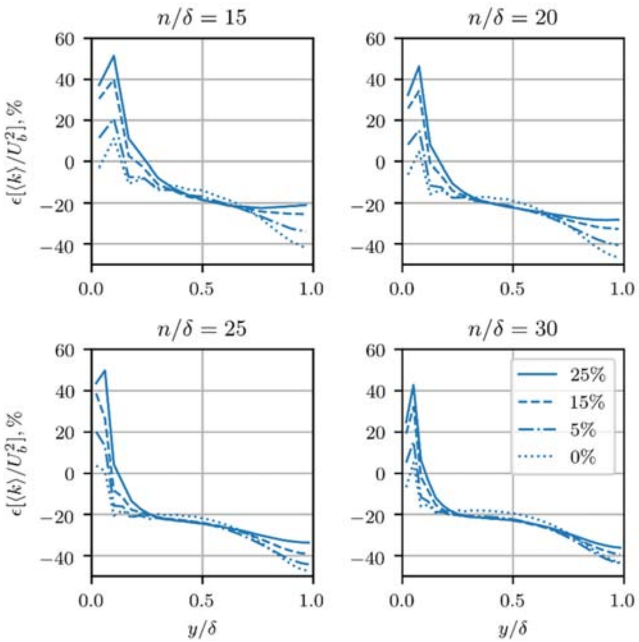

Fig. 3. Relative error in the outer-scaled mean turbulent kinetic energy profiles as a function of y/δ

Fig. 3 shows the relative errors in the profiles of the outer-scaled mean turbulent kinetic energy, $\langle k\rangle/U_b^2$. Firstly, it is evident that the magnitude of the errors is significantly higher than that observed for $\langle u\rangle$ and $\langle u_\tau\rangle$. Near the wall, a large error peak is observed, the size of which monotonously varies with the amount of upwinding. Above the overlap region, the values underpredict the reference DNS data instead. It is noted that the true values of $\langle k\rangle$ exhibit a large peak in the buffer layer, meaning that while the relative errors in the inner and outer regions are comparable in magnitude, the absolute error in the inner region is significantly larger. Interestingly, with increased resolution, the influence of upwinding appears to diminish in the outer region, leading to slightly improved accuracy for the cases of 0% and 5% upwinding. However, similar to the other analyzed flow quantities, no convergence with respect to n/δ can be observed.

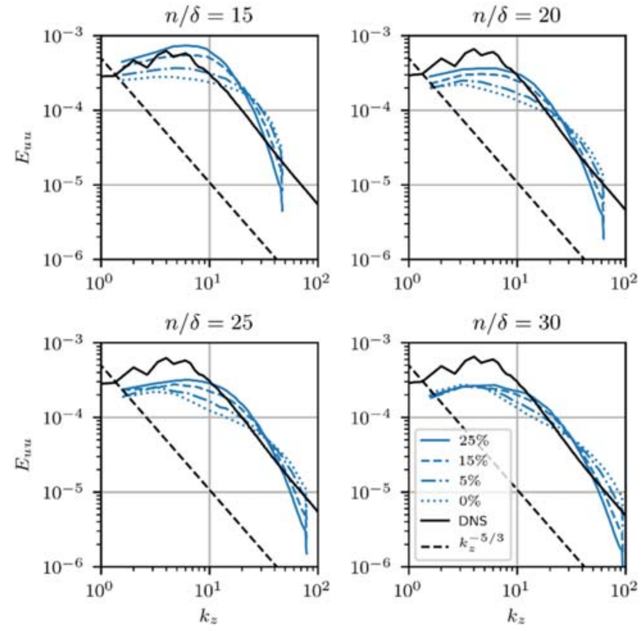

Fig. 4. Spanwise one-dimensional energy spectrum of u *at $y/\delta = 0.1$ (the location of the sampling point)*

Finally, the spanwise energy spectrum of the streamwise velocity is analyzed. In Fig. 4, the spectra at the location of the sampling point of the wall model, $y = 0.1\delta$, are presented. As expected, increased upwinding leads to faster damping of the high-frequency modes. A more surprising result is that it also leads to increased energy in the low-frequency motions. This effect is amplified at lower mesh resolutions. The inertial range (where the energy is expected to follow the $k_z^{-5/3}$ slope) is significantly less pronounced in the WMLES spectra compared to the DNS data. This is in part due to the limited frequency bandwidth in the WMLES and in part due to the damping of the high-frequency motions mentioned above.

5. Conclusions

Sixteen simulations of fully-developed turbulent channel flow have been conducted in order to investigate the effect of numerical dissipation on the predictive accuracy of wall-modelled LES. The amount of dissipation was varied by changing the resolution of the grid and the amount of upwinding used in the numerical scheme for interpolating convective fluxes at the cell faces.

Several trends in the error patterns of the considered flow quantities with respect to the amount of dissipation were identified. One observation, already reported in previous studies, is that results do not significantly improve with increased mesh resolution. This is unsatisfying, but it is important to keep in mind that mesh convergence is generally not well defined for LES with implicit filtering, and that the cell size plays the double role of defining the smallest eddies that can be resolved and controlling the amount of numerical dissipation. As indicated by the results, the latter affects the solution in a non-trivial way. A positive effect of higher resolution has been observed, however: The simulations using different amounts of upwinding resulted in more similar profiles. It is plausible that further improvements can be achieved by considering denser meshes than those covered in the simulation matrix, and simulations on such meshes is a subject of future works.

With respect to the dissipation coming from the linear upwind interpolation scheme, a monotonous change in the error patterns has been observed. An expected consequence of using more dissipative schemes is the damping of higher-frequency fluctuations, as reflected in the plots of the energy spectra (see fig. 4). A more interesting finding is that dissipation appears to lead to more energetic large-scale motions. These are likely connected to the observed near-wall peaks in the error in $\langle k \rangle$ (see fig. 3). These peaks were reported for the case of 25% upwinding before, and here it is shown that their size decreases monotonously with decreased upwinding. The mechanism behind the production of excessive $\langle k \rangle$ and the overly energetic large-scale motions requires further investigation and is a subject of future work. In particular, it would be interesting to analyze the budgets of k and each of the Reynolds stress components.

The mean velocity is the quantity for which increased dissipation led to increasingly more accurate results. It should be noted that the disparity among the error curves corresponding to different amounts of upwinding is only $\approx$ 1-2%, an order of magnitude less than what is observed for $\langle k \rangle$. It can be argued that the simulation parameters should be adjusted based on the more sensitive quantity. On the other hand, in practical applications the mean velocity is often seen as the primary quantity of interest, and to a large extent it also controls the error in the predictions of the wall shear stress made by the wall model (see fig. 1). Therefore, it is important to further develop the computational methodology so that an accurate mean velocity profile can be achieved without compromising second-order statistics.

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