DOI: 10.15514/ISPRAS-2022-34(5)-12

Overapproximation of the Number of Active Timers in Timed-Arc Petri Nets Using DP-Systems

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Abstract. Timed-arcs Petri nets are a time extension of Petri nets that allows assigning clocks to tokens. System of dynamic points on a metric graph (DP-systems) is another dynamical model that is studied in discrete geometry dynamics; DP-system combines continuous time and discrete branching events and used, for example, in study of localized Gaussian wave packets scattering on thin structures. In recent works, asymptotic estimates of the growth of the number of points in dynamic systems on metric graphs were obtained. In this paper, we provide a mean to overapproximate the number of different values of timers for a subclass of timed-arc Petri nets by constructing a system of dynamic points on a metric graph and prove overapproximation of the number of timer values by the number of points in the system of dynamic points.

Keywords: metric graphs; timed-arc Petri nets; dynamic properties

For citation: Dworzanski L.W. Overapproximation of the Number of Active Timers in Timed-Arc Petri Nets Using DP-Systems. Trudy ISP RAN/Proc. ISP RAS, vol. 34, issue 5, 2022. pp. 183-194. DOI: 10.15514/ISPRAS-2022-34(5)-12

Acknowledgements. The reported study was funded by RFBR according to the research project 20-07-01103a.

Оценка сверху числа активных таймеров в сетях Петри с временными дугами с помощью динамических систем точек на графах

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Аннотация. Сети Петри с временными дугами – это временное расширение сетей Петри (TaPN-сети), которое позволяет присваивать таймеры фишкам. Система динамических точек на метрическом графе (DP-система) это другая динамическая модель, которая рассматривается в теории геометрических дискретных динамических систем и, исторически, ее изучение мотивировано изучением распространения локализованных гауссовых волновых пакетов по тонким структурам. DP-система моделирует дискретные ветвящиеся события происходящие в реальном времени. В недавних работах были получены асимптотические оценки на рост числа точек в DP-системах на метрических графах. В данной работе мы предлагаем методы оценки сверху числа различных значений таймеров для подкласса сетей Петри с временными дугами с помощью построения DP-системы на метрическом графе по сети Петри и показывает, что количество различных значений таймеров в исходной сети Петри не превосходят количество точек в DP-системе. Это позволяет переносить известные оценки для DP-систем на сети Петри с временными дугами для выделенного подкласса.

Ключевые слова: метрические графы; сети Петри с временными дугами; динамические свойства

Для цитирования: Дворянский Л.В. Оценка сверху числа активных таймеров в сетях Петри с временными дугами с помощью динамических систем точек на графах. Труды ИСП РАН, том 34, вып. 5, 2022 г., стр. 183-194. DOI: 10.15514/ISPRAS-2022-34(5)-12

Благодарности: Работа выполнена при поддержке гранта РФФИ 20-07-01103 а.

1. Introduction

Petri nets are widely-used to model the behaviour of distributed concurrent computer systems and concurrent processes in biology, chemistry, physics, and other fields [1]. There are many time extensions of Petri nets were suggested [2, 3]. Simultaneously, almost any of semantical extensions makes Petri nets Turing-complete and many of general behavioral problems immediately become undecidable by Rice-Uspensky theorem. The widely known time extensions of Petri nets – Time Petri nets and Timed (Duration) Petri nets – are Turing-complete as they admit urgency and allow to model unbounded counters. Time extensions of Petri nets, as well as other real-time models, are under active study as, for many real-world software/hardware systems, time related aspects like performance, time-outs, delays, and latency are crucial for correct functioning [4-6]. A time semantics with restricted urgency was recently suggested for TaPN-nets in [7]; the suggested semantics allows urgent transitions to consume tokens only from the bounded places of a Petri net, and this restriction makes some behavioral problems decidable for TaPN-nets.

Timed-arc Petri nets (TaPN-nets) are an extension of Petri nets with real-time semantics: tokens are assigned clocks [8]; the inscription on an incoming arc of a transition define tokens of which age can be consumed by the firing of the transition.

TaPN-nets could be used to model mobile agents moving among nodes or data packets being transmitted between nodes of a telecommunication system. The number of tokens with different timers corresponds to the number of different data packets in a system. Examples of systems where the number of different packets is an important characteristic – networks of self-driven cars, high-loaded telecommunication systems experiencing DDoS attack, underwater network of drone swarm with intensive communication. Excessive number of packets stipulates communication quality degradation or failure. Therefore, to assess the quality of system functioning, it is important to estimate the number of tokens-packets with different timer values. In this paper, we suggest to an approach to approximate the number of different timers in a TaPN by the number of dynamic points on metric graphs.

Metric graph is a graph with lengths assigned to edges. Some results towards dynamical characteristics of systems of dynamic points flowing along undirected edges of a metric graphs in both directions were recently obtained [9-13]. In such systems, when a point reaches a vertex of the graph, new points start moving along all the edges incident to the vertex. The growth of the number of points moving along edges and its asymptotics were studied in [11, 12] and, for some special cases, in [14].

The orientation of metric graph edges allows us to approximate TaPN-nets more precisely; thus, in [15], the asymptotics of points number growth were obtained for systems of dynamic points on directed Sperner graphs. And later in [16], it was extended to directed Hamiltonian graphs.

The suggested simulation of a TaPN-net with a metric graph enables us to overapproximate the number of different clock values in the TaPN-net. A point in a constructed metric graph represents a clock value in the TaPN-net, and the event of the arrival of a point to a vertex simulates that the clock has reached a specific value. This allows us to estimate the number of different time events in a system modelled using TaPN-net. Translation to less expressive but more amenable to behavioural analysis or better studied models can be found in coverability analysis of Petri nets using Karp-Miller trees or in performance analysis of continuous timed Petri nets using Markov processes [17].

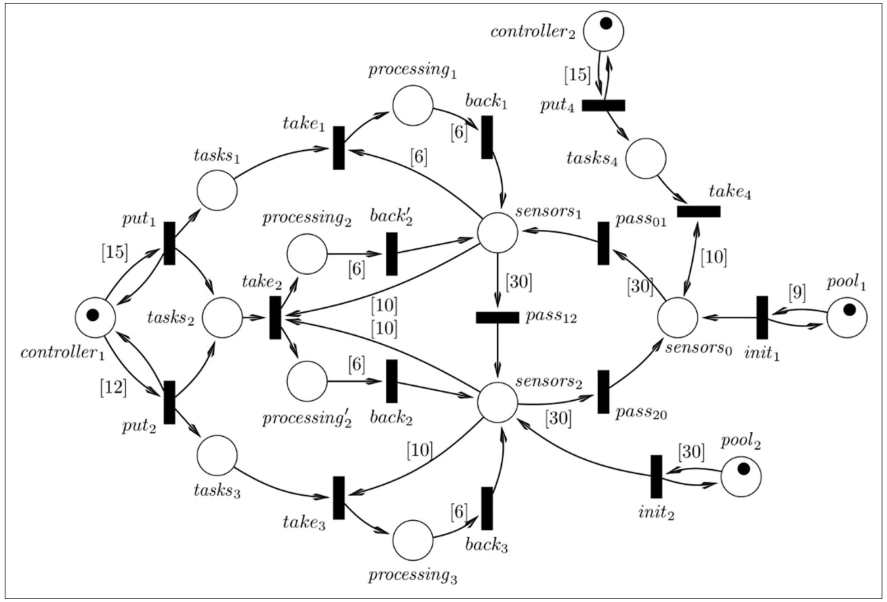

Fig. 1. TaPN-net model of an underwater wireless sensor network

The frequency of time events and its growth can be crucial as it determines energy or traffic load of the system. The growth of frequency can be important for such systems as software defined networks when resource management is adaptive [18] or when communication is complicated due to media. As a motivational example, we can consider a TaPN-net model of an underwater wireless sensor networks [19, 20]. The communication among underwater sensors is complicated due to the transmission medium (salt water); energy consumption and packets collisions impede efficient implementation of acoustic underwater transmission technology. Consider TaPN-net model $TaPN_{uwa}$ of an underwater acoustic sensor network (UWA-SN) in Figure 1. Places $sensors_{1\text{-}3}$ model spatial locations of underwater sensors, which measure temperature and pH levels. Sensors are regularly issued to places $sensors_0$ and $sensors_2$ from pools $pool_1$ and $pool_2$. Each 30 time units, a sensor flows to another spatial position. As communication is complicated and energy is an important resource, the active participation of sensors in communication shall be reduced to minimum. Thus, sensors receive commands to probe the water from controllers $controller_1$ and $controller_2$. Signal emission of controllers is modelled with places $tasks_{1\text{-}3}$. If a sensor is ready and receives a signal $tasks_{1\text{-}3}$, then it takes $take_{1\text{-}3}$ an assignment, processes $processing_{1-3}$ it, and returns $back_{1–3}$ to waiting states $sensors_{1–3}$.

A collision in a node may occur when several signals reach the node simultaneously. Such collisions depend on the spatial position of nodes in UWA-SN. When such a collision occurs, the node signals emitters about the problem, and the emitters cooperate to avoid collisions. Thus, the appearance of a new time event can result in additional traffic in the network. When many collisions occur, such reconfiguration can impede the normal functioning; therefore, the growth rate of the number of points could be an important parameter of the systems [19, 20].

To overapproximate the growth of different time event in the system TaPN , we build a metric graph Γ. The growth rate of points in Γ is not lower than thus of the number of different token-clock values in the TaPN-net. Thus, if the growth rate of Γ is acceptable, then so is the growth rate in TaPN.

Both models embrace real-time dynamics of discrete entities moving within a structure defined by a graph. We establish correspondence between a subclass of TaPN-nets and DP-systems. In Section

2, the notions of dynamical systems of points on metric graphs and timed-arc Petri nets are given. In Section 3, we provide the notion of overapproximation and supporting translation from a TaPN-net to a metric graph. In section 4, we provide a translation of a DP-system to a TaPN-net. Section 5 concludes the paper with some discussion on further research.

2. Preliminaries

By N, $Q_{\geq 0}$, R, we denote the sets of natural, non-negative rational, and real numbers, respectively. The set of open and closed intervals over $R_{\geq 0} \cup \{\infty\}$ is denoted by $I(R_{\geq 0})$. For a set *S*, a *bag* (*multiset*) *m* over *S* is a mapping $m : S \to N$. The set of all bags over *S* is denoted by N^S. We denote addition and subtraction of two bags by + and −, the number of all elements in *m* taking into account the multiplicity by $|m|$, and pointwise comparisons of bags by =, <, >, ≤, ≥, that are defined as $m_1Rm_2 \equiv \forall s \in S : m_1(s)Rm_2(s)$ where *R* is one of =, <, >, ≤, ≥. We overload the set notation writing ∅ for the empty bag and ∈ for the element inclusion.

2.1. DP-systems on directed metric graphs

A directed metric graph *Γ* is a graph consisting of set of vertices *V*, set of directed edges (arcs) *E*, and length function *l* mapping each arc $a = <v_1, v_2> \in E$ to a positive real, *i.e.*, $l: E \to R_+$ [21].

The arc opposite to arc $a = <v_i, v_j>$ is denoted by $\underline{a}$, *i.e.*, $\underline{a} = <v_j, v_i>$. For two points *x* and *y* on the graph, metric *ρ(x, y)* is the shortest distance between them, where distance is measured along the arcs of the graph additively. A walk is a finite or infinite sequence of arcs which joins a sequence of vertices.

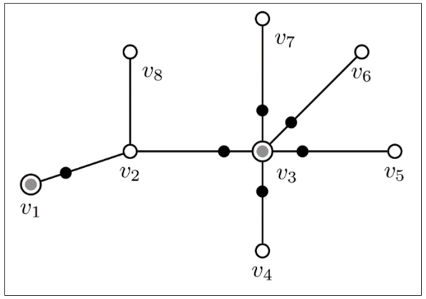

Fig. 2. System of dynamic points $P_Γ$ on metric graph Γ

Definition 1 (System of dynamic points). *A system of dynamic points (DP-system) on a metric graph $P_Γ$ is a pair <Γ, P>, where Γ is metric graph and P is a set of points distributed on vertices and edges of Γ.*

Dynamics of a DP-system $P_Γ$ is defined as following. In the initial state, a set of points is distributed on the arcs and vertices of *Γ*. The position of point *p* on arc *a* is denoted by $x_a(p)$ or just *x(p)*. When time starts to flow, each point *p* moves along its arc *a* direction. Each point *p* located at vertex *v*, for each outgoing arc *a* incident to *v*, produces a new point p_0 on each *a*, and *p* disappears (intuitively, this corresponds to wave packet scattering); each produced point p_0 starts moving along corresponding *a*. All points move with the same constant velocity; and, due to new points generation, some arcs may carry more than one point. When a moving point reaches the end incident to vertex v_0, again, on each outgoing arc incident to v_0, a new point is generated. When more than one points reach a vertex simultaneously at *t*, on each outgoing arc, only one point is produced, as if only one point has reached the vertex at *t*; *i.e.*, points met on a vertex fuse, and each coordinate of an arc can carry only one dynamic point.

In Figure 2, the initial set of points consists of two points in vertices v_1 and v_3. The point at v_1 produces a new point on edge $<v_1, v_2>$. The point at v_3 produces points on edges leading to vertices

v_2, v_4, v_5, v_6, v_7. After a time unit, there are no points in v_1 and v_3 (coloured grey), but there are points (coloured black) moving from v_1 and v_3 to their adjacent vertices.

The number of dynamic points in P_Γ at time t is denoted by $N_{P\Gamma}(t)$. The number of points on edge e at time t is denoted by $N_e(t)$. For dynamical systems of points P_Γ and $P'_{\Gamma'}$, we say that the growth rate of P_Γ is equal or less than thus of $P'_{\Gamma'}$ if $\forall t \in \mathbb{R}_+ \setminus Coll$: $N_{P\Gamma}(t) \leq N_{P'\Gamma'}(t)$, where $Coll$ is the (countable) set of time points when more than one dynamic point meet on a vertex; we exclude such time points as technically the number of dynamic points decreases for these moments. In what follows, we discuss number of points implicitly omitting vertices collision time points.

2.2. Timed-Arc Petri nets

Petri nets is a classical well-known formalism for concurrent systems modelling. A place/transition net (PT-net) is a Petri net with black tokens, that are indistinguishable from each other.

Definition 2 (Place/transition nets). *A PT-net is a tuple $<P,T,F,\gamma>$, where*

- *P and T are disjoint finite sets of places, respectively, transitions;*
- *$F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs (flow relation);*
- *$\gamma : F \rightarrow N$ is a weight function.*

For an element $x \in P \cup T$, an arc $<y,x>$ is called an *input arc*, and arc $<x,y>$ – an *output arc*. *Preset* $^\bullet x$ and a *postset* $x^\bullet$ are subsets of $P \cup T$ such that $^\bullet x = \{y | <y,x> \in F\}$ and $x^\bullet = \{y | <x,y> \in F\}$. A *marking* of N is a function m: $P \rightarrow \mathbb{N}$. A pair $<N,m>$ of a PT-net and a marking is called a marked net.

Let $N = <P,T,F,\gamma>$ be a PT-net. A transition $t \in T$ is *enabled* in a marking m iff $\forall p \in {}^\bullet t \Rightarrow m(p) \geq \gamma <p,t>$. An enabled transition t can *fire* yielding a new marking $m'(p) = m(p) - \gamma <p,t> + \gamma <t,p>$ for each $p \in P$ (denoted $m \rightarrow^t m'$). The set of all markings reachable from a marking m is denoted by *R(m)*.

Now, we provide the definition of Timed-Arc Petri nets (TaPN-nets) with token-based time semantics and urgency [6, 8, 22].

Definition 3 (Timed-Arc Petri net with Urgency). A TaPN-net is a tuple TaPN = $<N,\gamma^t,U>$, where

- *$N = <P,T,F,\gamma>$ is a PT-net called the skeleton of TaPN and denoted by S(TaPN);*
- *γ^t: $P \times T \rightarrow I(\mathbb{R}_+ \geq 0)$ is a set of token-age constraints on arcs;*
- *$U : T \rightarrow Q \geq 0$ is a set of urgency constraints on transitions.*

The marking $m = <m_s,m_t,m_u>$ of a TaPN-net $TaPN$ consists of a marking m_s of $S(TaPN)$, a time marking m_t: $Tok(m_s) \rightarrow \mathbb{R}_{\geq 0}$ that assigns clocks to tokens, and an urgency marking m_U: $T(m_s) \rightarrow \mathbb{R}_{\geq 0}$ that assigns clocks to transitions, where $T(m_s)$ comprises all enabled transitions and $Tok(m_s)$ comprises all tokens of the marked PT-net $<S(TaPN),m_s>$. The urgency constraint $U(t)$ means that t must fire if t has been enabled for $U(t)$ units of time. The token-age constraint $\gamma^t(p,t)$ defines that a firing of t may consume only token z in p with $m_t(z) \in \gamma^t(p,t)$. The urgency U of a transition is depicted as a number near the transition. The time constraints γ^t of an arc are depicted as an interval on the arc.

The operational semantics of TaPN-nets is defined by incorporating time constraints into the firing rules of PT-nets. Transition t is enabled in the marking $m = <m_s,m_t,m_u>$, if t is enabled in $<S(TaPN),m_s>$ and time constraints of t are satisfied, *i.e.*, each token α from a place p involved in the firing of t satisfies $m_t(z) \in \gamma^t(p,t)$.

An execution of a TaPN-net is a sequence of steps of the following two kinds.

A *transition firing step* is the firing of enabled transition t that consumes involved tokens from places $\bullet t$ and produces new tokens to places $t\bullet$; the urgency clock of t is set to zero, $m_u(t) = 0$, and, for each produced token α, the clock value is set to zero, *i.e.*, $m_t(\alpha) = 0$.

A *time elapsing* step corresponds to the elapsing δ time units in each clock of the marking m. We assume that all token clocks and transition clocks run at the same pace. We denote by $m+\delta$ the

marking with all its clocks increased by δ, *i.e.*, for each token $\alpha \in m_s$: $(m_t + \delta)(\alpha) = m_t(\alpha) + \delta$, and for each transition t : $(m_u + \delta)(t) = m_u(t) + \delta$. Under urgency restrictions, a time elapsing step δ is allowed if there are no $\delta' \in [0,\delta)$ such that the $m + \delta'$ marking has urgent transitions.

3. Overapproximation of the number of active timers

Coverability analysis or approximation using continuous Petri nets are classical examples of overapproximation in Petri net analysis. TaPN-nets with urgency are Turing-complete, and DP-systems on metric graphs are, obviously, less expressive (consider DP-systems dynamics monotonicity) [23]. Thus, we consider a restricted class of TaPN-nets, such that token-age constraints on input arcs are positive intervals of zero length and each transition t has zero urgency, *i.e.*, $U(t) = 0$.

In metric graphs, if points are met in a vertex simultaneously, they are coalesced, *i.e.*, the resulting effect is the same as if only one point came to the vertex. This peculiarity of metric graphs functioning hinders direct modelling of TaPN-nets using metric graphs. However, under the provided restrictions on TaPN nets, it is possible to overapproximate the number of different timers present simultaneously in a TaPN-net.

Let TaPN be a marked TaPN-net and $m = <m_s,m_t,m_u>$ be its marking. Let us introduce an equivalence relation $\approx_T$ over tokens:

$$\forall \alpha_1, \alpha_2 \in Tok(m): \alpha_1 \approx_T \alpha_2 \leftrightarrow \alpha_1 \in m_s(p) \wedge \alpha_2 \in m_s(p) \wedge m_t(\alpha_1) = m_t(\alpha_2)$$

where tokens α_1 and α_2 are $\approx_T$-equivalent iff both are located in the same place p and have the same clock values.

When the clock value of a token α located in place p passes the maximum value of all the time constraints on outgoing arcs of p, it cannot further involve in firings and its clock becomes redundant. Such tokens are dead [16] by time restrictions and we may remove their clocks from consideration. We denote by *ActiveTokens(m)* the set of active tokens in marking m

$$ActiveTokens(m) = \{\alpha \in Tok(m) \mid \alpha \in m_s(p) \wedge m_t(\alpha) < max_{\forall <p,t> \in F}(\gamma_t <p, t>)\}$$

The quotient set $Timers(m) = ActiveTokens(m)/\approx_T$ consists of the classes of live tokens in marking m; each equivalence class corresponds to a set of tokens located in a place p that have the same clock value. In a physical system, a set of agents represented by such a class can share the same timer/external signal. The cardinality of $|Timers(m)|$ is the number of timers simultaneously active in the system.

Let $TaPN = <<P, T, F, \gamma>, \gamma^t, U>$ be a TaPN-net, where $P = \{p_1 ... p_n\}$ and $T = \{t_1 ... t_m\}$, and let m_0 be its initial marking. We construct DP-system $P_\Gamma(TaPN)$, which will be used to overapproximates $TaPN$, with simultaneously constructing relation $\varphi_b \subset Timers(m) \times P_\Gamma$ between timers $Timers(m)$ and points in $P_\Gamma(TaPN)$ created to approximate these timers.

We start by adding a vertex v_i to $P_\Gamma(TaPN)$ for each transition $t_i \in T$. In Figure 3, an example of TaPN-net $TaPN$ and $P_\Gamma(TaPN)$ that overapproximates $TaPN$ is given. Then, for each pair of transitions t_i and t_j in T and place p_k in P such that there are edges $<t_i, p_k>$ and $<p_k, t_j>$ in F, we add an arc $<v_i, v_j>$ to $P_\Gamma(TaPN)$ with length $l(<v_i, v_j>) = \gamma_t(<p_k, t_j>)$. For each place p_k of P, for each token α in m_0 located in p_k, for each two transitions t_i and t_j, such that arcs $<t_i, p_k>$ and $<p_k, t_j>$ are in F, we put a new point p at the beginning of arc $<v_i, v_j>$ in $P_\Gamma(TaPN)$ and add $<[\alpha]_{\approx T}, p_i>$ to φ_b. Technically, we don't put p to v_i but on $<v_i, v_j>$ as putting p to v_i would produce tokens to all outgoing arcs. For each marked place p_k in P that has only outgoing arcs, for each transition t_j in T adjacent to p_k, we introduce a new vertice v_0 and a new arc $<v_0, v_j>$ into $P_\Gamma(TaPN)$ with length $l(<v_0, v_j>) = \gamma^t(<p_i, t_j>)$; we put a point p to v_0 (as for p_1 and t_1 in Fig. 3) and add $<tm, p>$ to φ_b, where $tm = <p_i, 0>$. The intention is that each potential firing of a transition t_j in $TaPN$, producing tokens to its output places, corresponds to an event when a point in $P_\Gamma(TaPN)$ reaches v_j, generating points on its outgoing arcs. In Figure 3, two active tokens oscillate between places p_2 and p_3 in $TaPN$, while their corresponding points in $P_\Gamma(TaPN)$ oscillate on edges between v_2 and v_3.

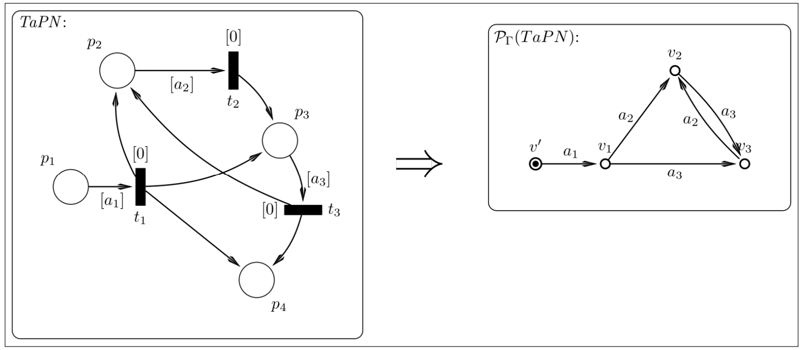

Fig. 2. TaPN-net TaPN and corresponding DP-system $P_\Gamma(TaPN)$

We say that P_Γ *overapproximates* TaPN-net *TaPN*, if for any *TaPN* execution $\sigma = m_0 \rightarrow^{s1} m_1 \rightarrow^{s2} m_2...$, for any state $m_i \in \sigma$ and global time $t \in R_+ \backslash Coll$ that corresponds to m_i, $|Timers(m_i)| \leq N_{P\Gamma}(t)$ holds.

Lemma 1. *Let TaPN be a TaPN-net with intervals of zero length as constraints on input arcs and each transition t has zero urgency. Let $P_\Gamma(TaPN)$ be a DP system constructed from TaPN. Then $P\Gamma(TaPN)$ overapproximates TaPN-net TaPN.*

Let $\sigma = m_0 \rightarrow^{s1} m_1 \rightarrow^{s2} m_2...$ be an execution of *TaPN*. For each m_i and corresponding time t, we define relation $\varphi_\tau \subset Timers(m_i) \times P_\Gamma$. For initial marking m_0 and time point $\tau = 0$, φ_0 is equal to φ_b. Let φ_t is defined for m_{i-1} and τ_{i-1} is global time corresponding to m_{i-1}. Let s_i be a time elapsing step for δ time units, and $\tau_i = \tau_{i-1} + \delta$ is the time corresponding to m_i.

For each place p_k, for each token $\alpha \in m_{i-1}$ located in p_k, for each two transitions t_l and t_j such that there are edges $<t_l, p_k>$ and $<p_k, t_j>$, if $(m_{i-1} + \delta)_t(\alpha) \leq \gamma_t(<p_k, t_j>)$, add $<t_m, p>$ to $\varphi_{\tau i}$, where point p is located on edge $<v_l, v_j>$ at coordinate $x(p) = (m_{i-1} + \delta)_t(\alpha)$ and $t_m = <p_k, (m_{i-1} + \delta)_t(\alpha)>$. If $(m_{i-1} + \delta)_t(\alpha) > \gamma_t(<p_k, t_j>)$, we don't add anything to φ_τ as it means that the age of α won't let it be involved in a firing of t_j.

Let s_i be a transition firing step for transition t_j and $\tau_i = \tau_{i-1}$ as time doesn't flow during a transition firing step. All pairs related to all tokens consumed by the firing are not more in φ_τ. For each new token $\alpha \in m_i$ in place p_k produced by the t_j-firing for each two transitions t_l and t_q such that there are edges $<t_l, p_k>$ and $<p_k, t_q>$, we add $<tm, p>$ to $\varphi_{\tau i}$, where point p located at the beginning of edge $<v_l, v_q>$, and $tm = <p_k, 0>$.

Note that for each marking m_i, for each active token α, image $\varphi_\tau([\alpha]_{\approx T})$ is not empty as, when all the pairs related to α cease from φ_τ, α is dead and $[\alpha]_{\approx T}$ ceases from $Timers(m_i)$ by definition. For different equivalence classes $tm_1 = <p_1, \tau_1>$ and $tm_2 = <p_2, \tau_2>$, images $\varphi_\tau(tm_1)$ and $\varphi_\tau(tm_2)$ are disjoint as they either lie on different graph edges if $p_1 \neq p2$, or lie on different coordinates if $\tau_1 \neq \tau_2$. Hence, $|Timers(m_i)| \leq N_{P\Gamma}(\tau)$.

Now let us consider a class of TaPN-nets without urgency and with inscriptions $[x, \infty)$ on input arcs of transitions. Now a token α may be involved in a firing of transition t, even if its clock passed maximal interval lower bounds at time τ and t is disabled. When some other token α_0 comes to an input place p of t and α_0 clock satisfies inscription on arc $<p, t>$, transition t may become enabled and may fire consuming α. Such a token α is not dead at τ but we call it passive. For such TaPN-nets, set of active tokens *ActiveTokens(m)* is defined as follows

$$ActiveTokens(m) = \{\alpha \in Tok(m) \mid \alpha \in m_s(p) \wedge m_t(\alpha) < max \{x \mid <p,t> \in F \;\&\; \gamma_t<p,t>=[x,\infty)\} \}$$

Theorem 2. *Let TaPN be a TaPN-net without urgency, with inscriptions $[x, \infty)$ on input arcs of transitions, and with initial marking m_0. Let $P_\Gamma(TaPN)$ be a DP-system constructed from TaPN. Then $P_\Gamma(TaPN)$ overapproximates TaPN-net TaPN.*

Let t be a transition and its firing produces tokens $\alpha_1...\alpha_n$ to output places of t. As inscriptions are unbounded, t may fire not necessary immediately at its earliest enabling time τ but later, at time $\tau_0 = \tau + \delta$. Due to the monotonicity of Petri net firing rule and monotonicity of time constraints of form $[x, \infty)$, we may fire transition t at time τ and then just keep (freeze) tokens $\alpha_1 ... \alpha_n$ at output positions of t until $\tau + \delta$. Such execution may only increase the number of tokens in *TaPN* at interval $[\tau, \tau + \delta]$. Henceforth, when we want to overapproximate execution σ we may consider execution σ' such that if a transition t will fire then it will fire at its earliest enabling time.

Now we may apply the similar argument as in Lemma 1, and obtain $|Timers(m_i)| \leq N_{P\Gamma}(\tau)$.

As urgency may only narrow a set of possible executions of a TaPN-net, we get the following corollary.

Corollary 3. *Let TaPN be a TaPN-net with urgency, with inscriptions $[x, \infty)$ on input arcs of transitions, and with initial marking m_0. Let $P_\Gamma(TaPN)$ be a DP-system constructed from TaPN. Then $P_\Gamma(TaPN)$ overapproximates TaPN net TaPN.*

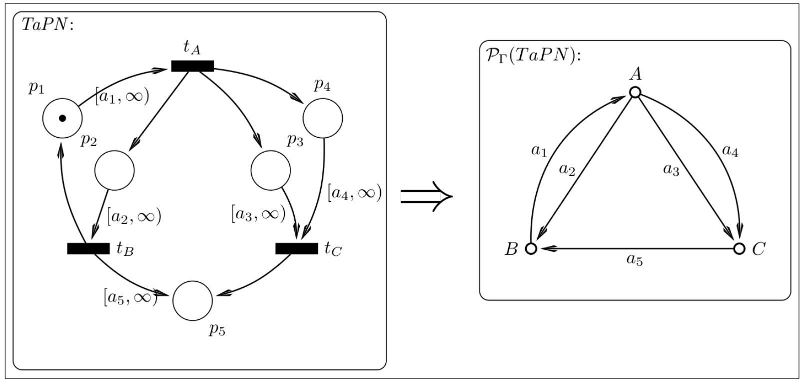

Fig. 4. TaPN-net TaPN and corresponding DP-system $P_\Gamma(TaPN)$

In Fig. 4, TaPN-net *TaPN* is on the left, and DP-system $P_\Gamma(TaPN)$ that overapproximates *TaPN* is on the right. In [7], it was calculated that the growth of the number of points in $P_\Gamma(TaPN)$ is

$$N(P_\Gamma(TaPN)) = t^2 (a_1 + a_2 + a_3 + a_4 + a_5) / (2(a_1 + a_4 + a_5) (a_2 + a_3) (a_1 + a_3 + a_5)) + O(t)$$

This gives us asymptotical estimate on the upper bound of the growth of number of active timers in TaPN.

In $P_\Gamma(TaPN)$, the arrival of a point to a vertex always corresponds to emitting of new points on the outgoing edges. In *TaPN*, the firing of transition t in *TaPN* may occur only when all input places $\bullet t$ have tokens with suitable clock values. Thus, some arrivals of points in Γ do not represent real firings of transition t in TaPN. In addition, tokens do not collapse in TaPN-nets; thus, we overapproximate the number of different clock values in TaPN. However, the number of different clock values is an important parameter for a system when, for example, its components use shared clocks.

4. Simulation of DP-systems using timed-arc Petri nets

To simulate DP-systems using Petri nets with real-time semantics, we need to represent points moving along edges of a graph using clocks in a Petri net. The advance of a point along an edge in a metric graph is modelled with the progress of the corresponding clock in a Petri net.

The number of points on a metric graph may grow indefinitely when edge lengths are incommensurable; therefore, it is not possible to model the evolution of a system of points on a metric graph using clocks in classical time or timed Petri nets [2, 24] because these models have finite structurally-determined number of clocks. On the contrary, in timed-arc Petri nets, clocks are assigned to tokens [25], and the number of clocks can grow along with the number of tokens in the net; therefore, TaPN-nets with token-based time semantics suits well for simulating DP-systems.

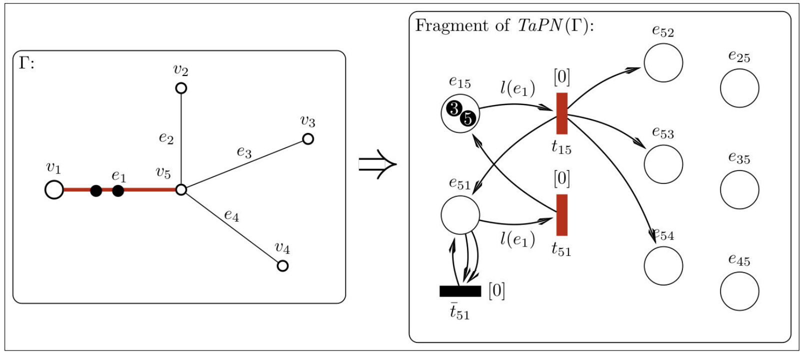

Fig. 5. A metric graph Γ and a fragment of TaPN(Γ) built from edge e_1

Let P_Γ be a DP-system and $TaPN(P_\Gamma)$ be the resultant TaPN-net simulating P_Γ. For each edge e of Γ connecting vertices a and b, we add two places e_{ab} and e_{ba}, where e_{ab} models arc $<a, b>$ and e_{ba} models arc $<b, a>$. In Fig. 5, DP-system P_Γ is on the left, and a fragment of its simulation $TaPN(P\Gamma)$ built for edge e_1 of Γ is on the right. Each point p on edge $<a, b>$ of P_Γ such that $x(p) = x_0$ is represented with a token α in p_{ab} such that $m_t(\alpha) = x_0$. In Fig. 5, two points on edge $<v_1, v_5>$ at the distances 3 and 5 from v_1 are modelled with two tokens that have timer values of 3 and 5, correspondingly. In addition, we add transitions t_{ab} and t_{ba} with $U(t_{ab}) = U(t_{ba}) = 0$, which firings model events when a point on e reaches b and a, respectively. We add arc $<e_{ab}, t_{ab}>$ with $\gamma t(<e_{ab}, t_{ab}>) = l(e)$, and, for each edge e_i in the metric graph connecting vertex b with vertex c, we add arc $<t_{ab}, e_{bc}>$ with $\gamma t(<t_{ab}, e_{bc}>) = 0$. Also, for each place e_{ab}, we add transition $\underline{t}^{ab}$ with $U(t^{ab}) = 0$ (as for e_{51} in Fig. 5); t^{ab} models collapsing of points in a vertex by consuming two tokens with clocks equals to 0 in e_{ab}, and puts only one token with a clock equal to 0 back.

The tool support and details of the translation software implementation are provided in [26]. The translation enables us to utilise well-known analysis tools for TaPN nets – TAPAAL/UPPAAL[29] and ReNew [30], to conduct behaviour analysis and numerical experiments for metric graphs. In Fig. 6, the translation of a metric graph DP to TaPN-net in TAPAAL representation using the developed tool is demonstrated.

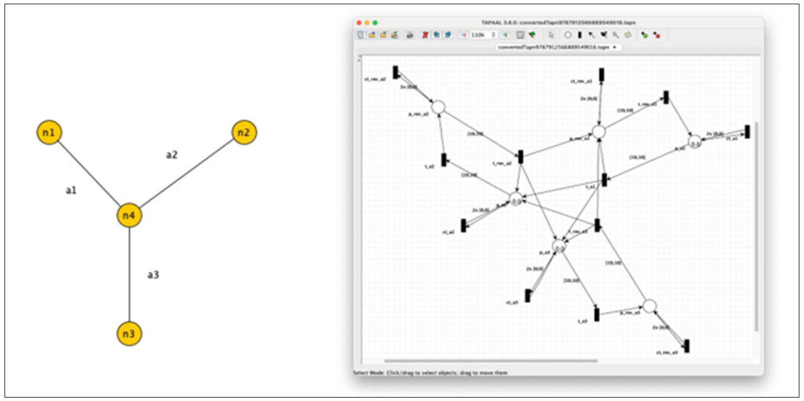

Fig.6. Tool support for translation of metric graph to TaPN-net

For example, the following properties can be checked using TCTL engine of TAPAAL:

– is it possible that more than 50 signals come to node X in 10 seconds;
– is it possible that in the next 5 minutes two messages come with the difference of their time moments less that ε milliseconds;
– does it hold, for the system initial phase with duration of 15 minutes, that if a point comes to node X, then no points comes to node Y within 10 seconds,

etc.

5. Conclusion

In this paper, we studied correspondence between metric graphs and a restricted class of TaPN-nets. We demonstrated that, for a restricted subclass of TaPN nets, it is possible to overapproximate the number of active timers using DP system on a metric graph. Such a correspondence connects studies on TaPN-nets and metric graphs, which allows to conduct analysis of the number of different timers in a given subclass of TaPN-nets using a system of points dynamics.

Our current research is to further weaken the suggested restrictions on TaPN-nets and, simultaneously, to find asymptotics for a more general class of DP-systems. Current working hypothesis is that the approach used to obtain estimates in [15, 16] could be extended to arbitrary directed graphs letting the suggested overapproximation to be used for a wider class of TaPN-nets. Conducted computed numerical experiments support it.

References / Список литературы

[1] Reisig W. Understanding Petri Nets - Modeling Techniques, Analysis Methods, Case Studies. Springer, 2013, 257 p.

[2] Berard B., Cassez F. et al. Comparison of different semantics for time Petri Nets. Lecture Notes in Computer Science, vol. 3707, 2005, pp. 293-307.

[3] Brown C., Gurr D. Timing Petri Nets categorically. Lecture Notes in Computer Science, vol. 623, 1992, pp. 571-582.

[4] Tvardovskii A.S., Yevtushenko N.V. On reduced forms of initialized Finite State Machines with timeouts. Trudy ISP RAN/Proc. ISP RAS, vol. 32, issue 2, 2020. pp. 125-134. DOI: 10.15514/ISPRAS-2020-32(2)-10.

[5] Tvardovskii A.S., Laputenko A.V. On the possibilities of FSM description of parallel composition of timed Finite State Machines. Trudy ISP RAN/Proc. ISP RAS, vol. 30, issue 1, 2018, pp. 25-40 (in Russian). DOI: 10.15514/ISPRAS-2018-30(1)-2 / Твардовский А.С., Лапутенко А.В. О возможностях автоматного описания параллельной композиции временных автоматов. Труды ИСП РАН, том 30, вып. 1, 2018 г., стр. 25-40.

[6] Akshay S., Genest B., Hélouët L. Decidable Classes of Unbounded Petri Nets with Time and Urgency. Lecture Notes in Computer Science, vol. 9698, 2016, pp. 301–322.

[7] Hanisch H.-M. Analysis of place/transition nets with timed arcs and its application to batch process control. Lecture Notes in Computer Science, vol. 961, 1993, pp. 282–299.

[8] Chernyshev V., Shafarevich A. Statistics of Gaussian packets on metric and decorated graphs, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 372, 2014, article id 20130145.

[9] Dworzanski L.W. Towards dynamic-point systems on metric graphs with longest stabilization time. arXiv preprint arXiv:2010.12528, 15 p.

[10] Chernyshev V.L. Tolchennikov A.A., Correction to the leading term of asymptotics in the problem of counting the number of points moving on a metric tree. Russian Journal of Mathematical Physics, vol. 24, issue 3, 2017, pp. 290–298.

[11] Chernyshev V.L., Tolchennikov A.A. The second term in the asymptotics for the number of points moving along a metric graph. Regular and Chaotic Dynamics, vol. 22, issue 8, 2017, pp. 937–948.

[12] Tolchennikov A.A., Chernyshev V.L., Shafarevich A.I. Asymptotic properties of the classical dynamical systems in quantum problems on singular spaces. Nonlinear Dynamics, vol. 6, issue 3, 2010, pp. 623-638 (in Russian)/ Толченников А.А., Чернышев В.Л., Шафаревич А.И. Асимптотические свойства и классические динамические системы в квантовых задачах на сингулярных пространствах. Нелинейная динамика, том 6, вып. 3, 2010 г., pp. 623-638.

[13] Chernyshev V. L., Tolchennikov A. A., Shafarevich A. I., Behavior of quasi-particles on hybrid spaces. relations to the geometry of geodesics and to the problems of analytic number theory, Regular and Chaotic Dynamics, vol. 21, issue 5, 2016, pp. 531-537.

[14] Chernyshev V., Tolchennikov A. Asymptotics of the number of endpoints of a random walk on a certain class of directed metric graphs. Russian Journal of Mathematical Physics, vol. 28, issue 4, 2021, pp. 434-438.

[15] Pyatko D., Chernyshev V. Asymptotics of the number of possible endpoints of a random walk on a directed hamiltonian metric graph. arXiv preprint arXiv:2112.13822, 2021, 15 p.

[16] Cohen G., Gaubert S., Quadrat J.-P., Asymptotic throughput of continuous timed petri nets. In Proc. of the 34th IEEE Conference on Decision and Control, vol. 2, 1995, pp. 2029-2034.

[17] Tuncer D., Charalambides M. et al. Adaptive resource man agement and control in software defined networks, IEEE Transactions on Network and Service Management, vol. 12, issue 1, 2015, pp. 18-33.

[18] Heidemann J., Stojanovic M., Zorzi M. Underwater sensor networks: applications, advances and challenges. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 370, 2012, pp. 158-175.

[19] Liu K., Yang Z. et al. Oceansense: monitoring the sea with wireless sensor networks. Mobile Computing and Communications Review, vol. 14, issue 2, 2010, pp. 7-9.

[20] Berkolaiko G., Kuchment P. Introduction to quantum graphs. Mathematical Surveys and Monographs, vol. 186. American Mathematical Society, 2013, 270 p.

[21] Bolognesi T., Lucidi F., Trigila S. From timed Petri Nets to timed LOTOS. In Proc. of the IFIP WG6.1 Tenth International Symposium on Protocol Specification, Testing and Verification X, 1990, pp. 377-406.

[22] de Frutos-Escrig D., Ruiz V.V., Alonso O.M. Decidability of properties of timed-arc petri nets. Lecture Notes in Computer Science, vol. 1825, 2000, pp. 187–206.

[23] Merlin P. A study of the recoverability of computer systems. Ph.D. Thesis, University of California, Irvine, CA, 1974, 154 p.

[24] Izmaylov A.A., Dworzanski L.W. Analysis of DP-systems Using Timed-Arc Petri Nets via TAPAAL Tool. Trudy ISP RAN/Proc. ISP RAS, vol. 32, issue 6, 2020, pp. 155-166. DOI: 10.15514/ISPRAS-2020-32(6)-12.

[25] David A., Jacobsen L. et al. TAPAAL 2.0: Integrated Development Environment for Timed-Arc Petri Nets. Lecture Notes in Computer Science, vol. 7214, 2012, pp. 492-497.

[26] Cabac L., Haustermann M., Mosteller D. Renew 2.5 – towards a comprehensive integrated development environment for Petri Net-based applications. Lecture Notes in Computer Science, vol. 9698, 2016, pp. 101–112.

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