DOI: 10.15514/ISPRAS-2026-38(3)-56

Software Implementation of the Cascade Calculation Method for Propeller Thrust

[1,2] *A.D. Sukhov, ORCID: 0000-0003-3527-3183 <sukhov.ad@phystech.edu>*
[1] *A.G. Petrov, ORCID: 0000-0002-0618-2285 <petrovipmech@gmail.com>*

[1] *Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, 101-1, Vernadskogo Av., Moscow, 119526, Russia.*

[2] *Ivannikov Institute for System Programming of the Russian Academy of Sciences, 25, Alexander Solzhenitsyn st., Moscow, 109004, Russia.*

Abstract. A software implementation of a multilevel method for calculating the force characteristics of propellers based on the generalized Zhukovsky profile and a cascade model of blade interaction is presented. The developed algorithm allows for the sequential calculation of aerodynamic characteristics from a single profile to a complete propeller blade, taking into account flow separation and the mutual influence of profiles in the grid. The method is based on an analytical solution to the problem of flow around a single profile without separation and a numerical scheme for determining separation points using the Kochin-Loitsansky criterion. Both similarity methods and a boundary element scheme with exponential convergence are used to calculate profile grids. The complete propeller is calculated using the cascade method with integration of aerodynamic characteristics along the blade radius. The developed software package reduces calculation time by approximately 10,000 times compared to CFD methods. The implemented modular architecture allows the results obtained to be used in optimization design systems and as input data for models such as the Actuator Line Model.

Keywords: propeller; aerodynamic coefficients; Zhukovsky profile; cascade method; grid method; flow separation; numerical methods; software implementation; high-speed calculation; boundary layer.

For citation: Sukhov A.D., Petrov A.G. Software implementation of the cascade calculation method for propeller thrust. Trudy ISP RAN/Proc. ISP RAS, vol. 38, issue 3, part 4, 2026, pp. 207-216. DOI: 10.15514/ISPRAS-2026-38(3)-56.

Acknowledgements. The study was funded by the Russian Science Foundation, project no. 23-71-10091.

Программная реализация метода каскадного расчёта тяги гребных винтов

[1,2] *А.Д. Сухов, ORCID: 0000-0003-3527-3183 <sukhov.ad@phystech.edu>*
[1] *А.Г. Петров, ORCID: 0000-0002-0618-2285 <petrovipmech@gmail.com>*

[1] *Институт проблем механики им. А.Ю. Ишлинского РАН, Проспект Вернадского, 101-1, Москва, 119526, Россия.*

[2] *Институт системного программирования им. В.П. Иванникова РАН, ул. Александра Солженицына 25., Москва, 109004, Россия.*

Аннотация. Представлена программная реализация многоуровневого метода расчёта силовых характеристик гребных винтов, основанного на обобщённом профиле Жуковского и каскадной модели взаимодействия лопастей. Разработанный алгоритм позволяет выполнять последовательный расчёт аэродинамических характеристик от одиночного профиля до полной лопасти винта, с учётом отрыва потока и взаимного влияния профилей в решётке. В основу метода положены аналитическое решение задачи обтекания одиночного профиля без отрыва и численная схема определения точек отрыва по критерию Кочина-Лойцянского. Для расчёта решёток профилей применяются как методы подобия, так и схема граничных элементов с экспоненциальной сходимостью. Расчёт полного винта осуществляется методом каскадов с интегрированием аэродинамических характеристик по радиусу лопасти. Разработанный программный комплекс обеспечивает снижение времени расчёта примерно в 10 000 раз по сравнению с методами CFD. Реализованная модульная архитектура позволяет использовать полученные результаты в составе систем оптимизационного проектирования и в качестве входных данных для моделей типа Actuator Line Model.

Ключевые слова: гребной винт; аэродинамические коэффициенты; профиль Жуковского; каскадный метод; метод решёток; отрыв потока; численные методы; программная реализация; быстродействующий расчёт; пограничный слой.

Для цитирования: Сухов А.Д., Петров А.Г. Программная реализация метода каскадного расчёта тяги гребных винтов. Труды ИСП РАН, том 38, вып. 3, часть 4, 2026 г., стр. 207–216 (на английском языке). DOI: 10.15514/ISPRAS–2026–38(3)–56.

Благодарности. Исследование было поддержано грантом РНФ № 23-71-10091.

1. Introduction

Calculating the power characteristics of propellers is traditionally considered one of the most computationally complex tasks in hydro-aerodynamics. Full-volume numerical methods based on solving Navier–Stokes equations (RANS, LES) provide high accuracy but require significant computing resources and calculation time, which limits their application in the optimization design stages. To analyze a large number of blade geometry options and evaluate characteristics at different operating modes, methods are needed that provide acceptable accuracy at significantly lower computational costs.

Classical analytical approaches, dating back to the works of N. E. Zhukovsky [1], A. M. Basin [2], L. I. Sedov [3], and L. G. Loitsyansky [4], allow closed expressions for circulation and pressure distribution on profiles to be obtained, but they do not take into account flow separation and do not describe the interaction of neighboring profiles in the grid. The development of these ideas in combination with numerical methods has made it possible to construct hybrid schemes that combine the analytical form of the profile with corrections based on the solution of boundary layer equations [5-7].

In recent years, a number of modifications to classical methods have been proposed, including the cascade method, based on the generalized Zhukovsky profile and the introduction of universal correction coefficients for calculating profile grids [8]. This approach allows us to move from local calculations of single profiles to an integral determination of the thrust and torque of the entire propeller. At the same time, the accuracy is maintained at 10–25% compared to experiments and CFD, and the calculation time is reduced by more than 10,000 times [8-10].

Between high-fidelity CFD approaches and simplified analytical methods, reduced-order models such as the Actuator Line Model (ALM) [10] are widely used in propeller and rotor simulations. ALM relies on precomputed aerodynamic characteristics of blade sections and distributes body forces into the flow field, significantly reducing computational cost while retaining essential three-dimensional effects. However, the accuracy and robustness of ALM critically depend on the quality and availability of sectional aerodynamic data. In this context, the proposed cascade-based method can be considered as an efficient supplier of aerodynamic characteristics for ALM and similar hybrid models, bridging the gap between purely analytical approaches and full CFD simulations.

The purpose of this work is to describe the software implementation of a multilevel method for calculating the force characteristics of propellers, including:

- calculation of aerodynamic coefficients of single profiles taking into account flow separation;
- simulation of profile grids using similarity and boundary element methods;
- integration of force characteristics along the blade radius using a cascade scheme;
- formation of a universal software architecture suitable for rapid calculation and visualization of results.

The developed software system is designed for use in parametric optimization tasks, as part of digital twins of propellers, and for accelerated evaluation of characteristics as part of Actuator Line Model.

2. Process logic

The input data are:

- propeller geometry (distribution of installation angles and chords along the radius);
- coordinates or digitization of blade profiles;
- physical parameters of the medium: density ρ, axial velocity V_{ax}, rotation frequency n;
- calculation mode: unsteady, considering separation, grid method, or cascade.

The program performs the following sequential steps.

2.1 Profile approximation

Based on the digitized coordinates, the Zhukovsky conformal mapping is performed.

Here, $z = x + iy$ denotes the complex coordinate in the physical plane, while Z represents the complex coordinate in the parameter plane:

$$\frac{z-z_0}{z+z_0} = \left(\frac{Z-Z_0}{Z+Z_0}\right)^{z_0/Z_0},$$

which ensures a smooth contour and analytical computability.

2.2 Calculation of a single profile

At this stage, the aerodynamic characteristics of an isolated blade section are evaluated for a given angle of attack θ. The airfoil geometry obtained at the previous step is treated as a two-dimensional profile in an incompressible viscous flow.

First, the pressure and velocity distributions along the airfoil surface are computed using an analytical potential-flow solution for the Zhukovsky profile. The complex velocity field is obtained from the derivative of the conformal mapping, which allows the surface pressure coefficient $C_p(x)$ to be evaluated along the chord. Based on the resulting pressure distribution, the sectional lift and drag coefficients $C_y(\theta)$ and $C_x(\theta)$ are calculated by integrating the pressure forces over the airfoil contour. In the absence of flow separation, this procedure yields smooth and physically consistent polars in the linear lift region.

To account for viscous effects and flow separation at higher angles of attack, an optional correction is applied using the SeparationSolver module. The separation points on the suction and pressure sides of the airfoil are determined according to the Kochin–Loitsyansky criterion [7], with additional stabilization based on the Howarth postulate. The pressure distribution downstream of the separation point is modified accordingly, which leads to a reduction in lift and an increase in drag in the post-stall regime.

The resulting aerodynamic coefficients $C_y(\theta)$ and $C_x(\theta)$, computed both with and without separation correction, serve as input data for subsequent cascade and blade-integration stages.

2.3 Calculation of the profile cascade

Two options are used:

- similarity method (using universal coefficients $\chi(\xi)$) [11];
- boundary element method (with exponential convergence according to Petrov [12]).

Let $C_y^{\text{solo}}(\theta)$ and $C_x^{\text{solo}}(\theta)$ denote the lift and drag coefficients obtained for a single profile. For a periodic cascade of the same profile and step h we introduce the non-dimensional correction factors

$$\chi_y\left(\frac{h}{c}\right)=\frac{C_y^{\text{cas}}}{C_y^{\text{solo}}},\qquad \chi_x\left(\frac{h}{c}\right)=\frac{C_x^{\text{cas}}}{C_x^{\text{solo}}},$$

where c is the profile chord length.

A dedicated parametric study has been carried out using the saturation-free boundary-element formulation [12] (including Γ correction by Howarth's postulate) for representative sections (NACA-6409 and a Ka4-70 blade sections) and for angles of attack in the range $-20° \le \theta \le +20°$. The results, summarised in Fig. 1., show that χ_y and χ_x collapse onto quasi-universal curves that depend almost exclusively on the relative step h/c. It is worth noting that the same result is obtained for the circulation ratio.

For the practically relevant interval the scaling

$$C_y^{\text{cas}}(\theta)=\chi_y(h/c)C_y^{\text{solo}}(\theta),\qquad C_x^{\text{cas}}(\theta)=\chi_x(h/c)C_x^{\text{solo}}(\theta),$$

permits a direct evaluation of cascade polars by a single multiplication, thus eliminating the need for the computationally demanding periodic-domain solution.

An empirical polynomial approximation for the cascade correction factor $\chi(h/c)$ was obtained based on numerical simulations within a practically relevant range of relative spacing. The approximation is valid for $0.5 \le h/c \le 3.5$ and should not be extrapolated beyond this interval:

$$\chi(\xi) = -0.0008\xi^6 - 0.087\xi^5 + 1.277\xi^4 - 7.098\xi^3 + 19.46\xi^2 - 26.68\xi + 15.88.$$

The validity of this approximation was verified using a representative set of airfoil geometries, including NACA 6409, symmetric NACA 0012–0018 profiles, as well as the blade section profiles of the considered propeller. For each profile, the cascade correction factors were evaluated over a range of angles of attack relevant to practical operating conditions. The obtained results indicate that, within the specified interval of relative spacing and angles of attack, the approximation provides consistent and physically reasonable corrections, largely independent of the specific airfoil shape. At the same time, the authors acknowledge that a more comprehensive assessment of the applicability limits would require extending the analysis to a broader class of profiles, Reynolds numbers, and cascade configurations, which is considered a subject of future work.

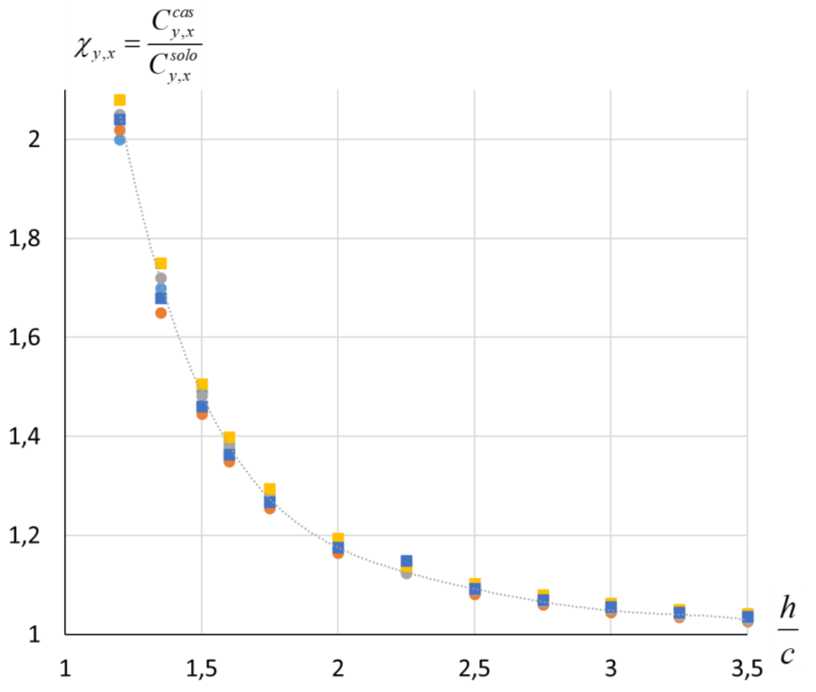

Fig. 1. Correction factors $\chi_y = C_y^{\text{cas}}/C_y^{\text{solo}}$ (circles) and $\chi_x = C_x^{\text{cas}}/C_x^{\text{solo}}$ (squares) as functions of relative spacing h/c. Marker legend: light-blue circle & blue square - Ka4-70 at ($r = 35\,mm$, $\beta_0 = 155°$); orange circle & yellow square - Ka4-70 at ($r = 11\,mm, \beta_0 = 125°$); grey circle - reference NACA-6409 ($\beta_0 = 130°$); line - polynomial fit. [11]

It should be emphasized that, due to the polynomial nature of the approximation, $\chi(h/c)$ exhibits unbounded behavior outside the specified range. In particular, for $h/c > 4.385$ the polynomial yields negative values, which have no physical meaning. Therefore, the approximation should be interpreted as a convenient engineering fit rather than a mathematically universal model.

2.4 Calculation of the complete propeller (cascade method)

The blade is divided into N sections with radii r_i.
For each section, the local flow parameters are calculated:

$$V_i = \sqrt{(\omega r_i)^2 + V_{ax}^2},$$
$$\varphi_i = \arctan \frac{\omega r_i}{V_{ax}},$$
$$\theta_i = \alpha_i - \varphi_i.$$

Then the forces on the element are found:

$$dT_i = \frac{1}{2}\rho V_i^2 [C_y(\theta_i)\cos\varphi_i - C_x(\theta_i)\sin\varphi_i] c_i, dr_i,$$
$$dQ_i = \frac{1}{2}\rho V_i^2 [C_y(\theta_i)\sin\varphi_i + C_x(\theta_i)\cos\varphi_i] c_i r_i, dr_i.$$

Integration over the radius is performed as

$$T = B\sum_i dT_i\,, \quad Q = B\sum_i dQ_i.$$

2.5. Construction of characteristics

The dimensionless thrust coefficient, torque coefficient, and efficiency are calculated:

$$K_T = \frac{T}{\rho n^2 D^4}, K_Q = \frac{Q}{\rho n^2 D^5}, \eta = \frac{J K_T}{2\pi K_Q}.$$

Next, the dependencies $K_T(J)$ and $\eta(J)$ are constructed.

3. Software implementation structure

The developed software implements a multi-level approach to calculating the power characteristics of propellers. The architecture is modular and allows for sequential steps to be performed–from profile approximation to blade radius integration (Table. 1).

Each calculation level has a separate class that implements specialized methods.

Table 1 General structure of the software.

Class	Main function	Related methods
`Airfoil`	storage of coordinates and approximation of the Zhukovsky profile	`approximate()`, `transform()`, `plot()`
`FlowSolver`	analytical calculation of pressure and velocity distribution along the profile	`calculate_cp()`, `integrate_cl_cd()`
`SeparationSolver`	determination of breakaway points using the Kochin–Loitsansky method, taking into account the Howarth criterion	`find_separation_points()`, `correct_cp()`
`Cascade`	calculation of correction factors χ_x, χ_y for the profile grid	`compute_chi()`, `apply_correction()`
`BladeSection`	storage of radial section parameters and local angles of attack	`evaluate_for_section()`
`Propeller`	integration of forces along the blade radius, obtaining K_T, K_Q, η	`integrate_thrust()`, `integrate_torque()`
`Visualizer`	plotting graphs $C_p(x)$, $C_y(\alpha)$, $K_T(J)$ and reporting data	`plot_cp()`, `plot_kt_kq()`

The overall computation workflow is organized as follows. First, the Airfoil class approximates the blade section geometry using the Zhukovsky transformation. Next, FlowSolver evaluates the pressure and velocity distributions and computes lift and drag coefficients for a given angle of attack. If enabled, SeparationSolver corrects these coefficients by accounting for flow separation. Cascade applies interaction corrections for a periodic blade grid. Finally, the Propeller class integrates sectional forces along the blade radius to obtain integral thrust, torque, and efficiency coefficients.

4. Interface and data output

The program generates:

- tables of pressure distributions and coefficients C_x, C_y;

- graphs $C_p(x)$, $C_y(\alpha)$, $K_T(J)$, $K_Q(J)$;
- a text report (CSV or JSON format) with the parameters of each section and the final characteristics of the propeller.

The results can be used:

- to evaluate the efficiency of the blade geometry;
- as input data for CFD models such as the Actuator Line Model;
- as part of the optimization design contours for propellers.

5. Results

For single profile, the aerodynamic coefficients $C_y(\alpha)$ and $C_x(\alpha)$ were computed for NACA 6409 profile in the range $-4° \leq \alpha \leq +12°$. The obtained curves show excellent agreement with experimental data [13], with a deviation not exceeding 10 % up to $\alpha = 8°$ (Fig. 2). When the flow separation module (SeparationSolver) is activated, the stall angle and the reduction of C_y after $\alpha \approx 16°$ are correctly reproduced in calculation (not shown on the plot).

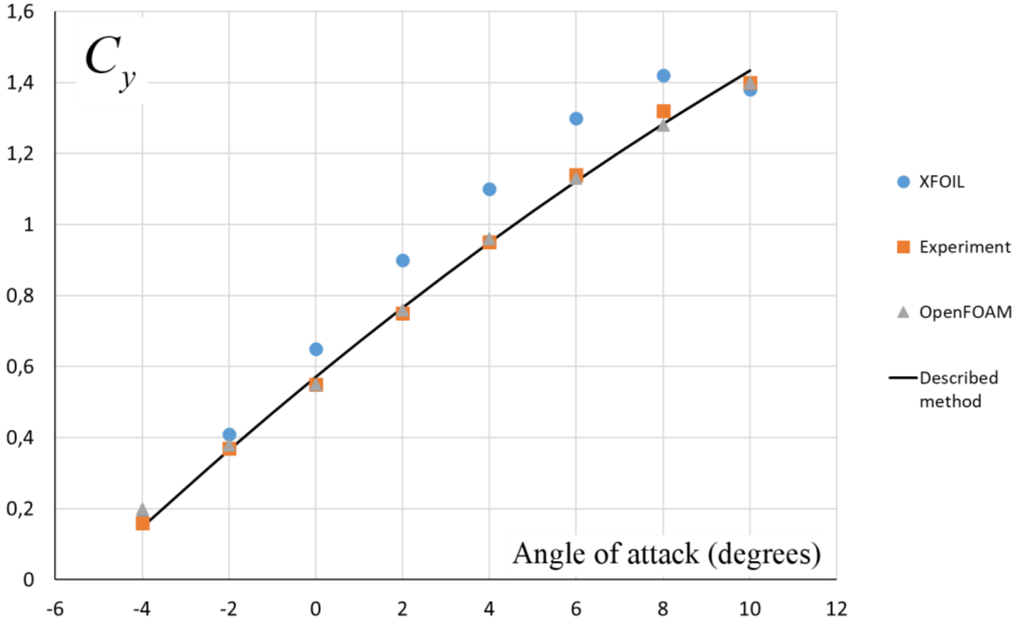

Fig. 2. Comparison of calculated and experimental $C_y(\alpha)$ for NACA-6409. Here, the parameter a denotes the angle of attack expressed in degrees.

For the complete propeller, calculations were performed for the Ka4-70 geometry ($D = 0.1$ m, $n = 500$ rpm) using the cascade method. The blade was divided into 10 radial sections. The calculation of the thrust coefficient showed $K_T = 0.59$, which is comparable to the result $K_T = 0.53$ in RANS and the OpenFOAM calculation. At the same time, the calculation using the method described above provides a time saving of approximately 10,000 times. All calculations can be performed on a standard laptop (Intel i7-1255U).

The results confirm that the developed method provides a reliable compromise between computational speed and physical accuracy. The use of analytical representations for the Zhukovsky profile and the Kochin–Loitsansky separation model ensures smooth behavior of force coefficients across the stall region, while the cascade correction allows for accurate evaluation of mutual blade interaction.

Although the approach does not resolve three-dimensional effects explicitly, the resulting error remains within 10% compared to CFD and experimental data, which is acceptable for preliminary and optimization design stages.

The modular implementation also makes it possible to embed the developed solver into higher-level frameworks such as digital twins of propellers, real-time control models, or actuator-line-based flow simulations. Thus, the presented method combines analytical transparency, numerical stability, and high computational performance, making it suitable for parametric optimization and conceptual design of marine propellers.

In the present study, the primary focus is placed on the thrust coefficient as a representative integral characteristic.

The extension of the validation to torque and efficiency coefficients, as well as to a broader set of propeller geometries, is planned as future work.

References

[1]. Жуковский Н. Е. Теория гребного винта с большим числом лопастей. Труды Отделения физических наук Общества любителей естествознания, 1907, т. 14, № 1, с. 1-40. / Zhukovsky N. E. Theory of a propeller with a large number of blades. Proceedings of the Physical Sciences Department of the Society of Naturalists, 1907, vol. 14, no. 1, pp. 1-40 (in Russian).

[2]. Басин А. М., Миниович И. Я. Теория и расчёт гребных винтов. Л., Судпромгиз, 1963, 400 с. / Basin A. M., Miniovich I. Ya. Theory and Calculation of Marine Propellers. Leningrad, Sudpromgiz, 1963, 400 p. (in Russian).

[3]. Седов Л. И. Плоские задачи гидродинамики и аэродинамики. М., Наука, 1966, 704 с. / Sedov L. I. Plane Problems in Hydrodynamics and Aerodynamics. Moscow, Nauka, 1966, 704 p. (in Russian).

[4]. Лойцянский Л. Г. Механика жидкости и газа. М., Наука, 1973, 840 с. / Loitsyanskii L. G. Fluid and Gas Mechanics. Moscow, Nauka, 1973, 840 p. (in Russian).

[5]. Петров А. Г. Аналитическая гидродинамика. М., Наука, 2010, 560 с. / Petrov A. G. Analytical Hydrodynamics. Moscow, Nauka, 2010, 560 p. (in Russian).

[6]. Petrov A. G., Sukhov A. D., Sibgatullin I. N. Laminar flow of viscous fluid around elliptical contours at an angle of attack. Lobachevskii Journal of Mathematics, 2022, vol. 43, no. 5, pp. 1184-1194. DOI: 10.1134/S1995080222050162.

[7]. Petrov A. G., Sukhov A. D., Britov A. D. Analytical and numerical methods for Zhukovsky airfoils aerodynamics coefficients. 2022 Ivannikov ISPRAS Open Conference (ISPRAS). IEEE, 2022, pp. 62-64. DOI: 10.1109/ISPRAS56028.2022.10021225.

[8]. Сухов А. Д., Петров А. Г. Расчёт силовых характеристик винта методом каскада. ИПМех РАН, ИСП РАН, 2024. / Sukhov A. D., Petrov A. G. Propeller power characteristics calculation by the cascade method. Institute for Problems in Mechanics RAS, Ivannikov ISP RAS, 2024 (in Russian).

[9]. Bergmann O., et al. Comparison and evaluation of blade element methods against RANS simulations and test data. CEAS Aeronautical Journal, 2022, vol. 13, no. 2, pp. 535-557. DOI: 10.1007/s13272-021-00580-2.

[10]. Britov A., et al. Numerical simulation of propeller hydrodynamics using the open source software. International Conference on Computational Science. Cham, Springer Nature Switzerland, 2023, pp. 245-257. DOI: 10.1007/978-3-031-36027-5_20.

[11]. Petrov A. G., Sukhov A. D. Numerical calculation of propeller force characteristics by the cascade method. Lobachevskii Journal of Mathematics, 2025 (in print).

[12]. Petrov AG. Saturation-free numerical scheme for computing the flow past a lattice of airfoils and the determination of separation points in a viscous fluid. Computational Mathematics and Mathematical Physics. 2011 Jul;51(7):1239-50.

[13]. Garcia, E. N., Durgesh, V., Johari, H. Experimental study of NACA symmetric and camber airfoils at low Reynolds numbers. 55th AIAA Aerospace Sciences Meeting. 2017.

Информация об авторах / Information about authors

Александр Георгиевич ПЕТРОВ – доктор физико-математических наук, ведущий научный сотрудник. Лауреат золотой медали имени С.А. Чаплыгина (2020). Автор и разработчик

численных схем метода граничных элементов с экспоненциальной сходимостью. Опубликовал около десяти работ в российских и зарубежных научных журналах по численным методам гидродинамики и вычислительной механике.

Alexander Georgievich PETROV – Dr. Sci. (Phys.-Math.), Leading Researcher. Laureate of the S.A. Chaplygin Gold Medal (2020). Author and developer of numerical boundary element methods with exponential convergence. Published about ten papers in Russian and international journals in the field of numerical hydrodynamics and computational mechanics.

Артем Дмитриевич СУХОВ – аспирант первого года обучения Института проблем механики Российской академии наук (ИПМех РАН). Автор публикаций по гидродинамике и применению метода граничных элементов в научных журналах ЖЭТФ и Лобачевский.

Artem Dmitrievich SUKHOV – first-year postgraduate student at the Institute for Problems in Mechanics of the Russian Academy of Sciences (IPMech RAS). Author of publications on hydrodynamics and applications of the boundary element method in JETP and Lobachevskii Journal of Mathematics.